

Critical local moment fluctuations in the Bose-Fermi Kondo model

Lijun Zhu and Qimiao Si

Department of Physics & Astronomy, Rice University, Houston, TX 77005-1892

We consider the critical properties of the Bose-Fermi Kondo model, which describes a local moment simultaneously coupled to a conduction electron band and a fluctuating magnetic field, i.e., a dissipative bath of vector bosons. We carry out an ϵ -expansion to higher than linear orders. (Here ϵ is defined in terms of the power-law exponent of the bosonic-bath spectral function.) An unstable fixed point is identified not only in the spin-isotropic case but also in the presence of anisotropy. It marks the point where the weight of the Kondo resonance has just gone to zero, and the local moment fluctuations are critical. The exponent for the local spin susceptibility at this critical point is found to be equal to ϵ in all cases. Our results imply that a quantum phase transition of the “locally critical” type is a robust microscopic solution to Kondo lattices.

PACS numbers: 75.20.Hr, 71.10.Hf, 71.27.+a, 71.28.+d

I. INTRODUCTION

The interest in quantum criticality in strongly correlated metals arises primarily because it provides a mechanism for non-Fermi liquid behavior. While the problem appears to be important for a number of correlated electron systems – including high temperature superconductors – the issues are particularly well-defined in heavy fermion metals [1,2,3,4,5,6,7,8,9,10,11,12]. Here, many materials have been shown to explicitly display a magnetic quantum critical point (QCP), including a growing list of stoichiometric or nearly stoichiometric ones [2,3,4,5]. A particularly striking puzzle has emerged from inelastic neutron-scattering experiments [13,14,15,16] and magnetization measurements [4,13,14] in some of these (nearly) stoichiometric materials. The dynamical spin susceptibility displays an ω/T scaling with a fractional energy/temperature exponent. In addition, the same exponent is seen not only at the antiferromagnetic ordering wavevector but also essentially everywhere else in the Brillouin zone.

In a recent work [17,18,19,20], a “locally-critical point” is found in Kondo lattice systems. This picture appears to explain the salient features of the aforementioned experiments. At a locally critical point, spatially extended critical fluctuations co-exist with spatially local ones. More microscopically, the criticality of the local Kondo physics is “embedded” into that associated with the magnetic ordering of the lattice: the divergence of the spatial correlation length is accompanied by the destruction of the Kondo resonance. The microscopic analysis [17,18] was carried out within an extended dynamical mean field theory (EDMFT), in which the Kondo lattice system is treated in terms of a single-impurity Bose-Fermi Kondo model supplemented by a self-consistency condition. The impurity model describes a local moment coupled at once to a conduction electron band and a dissipative bath of vector bosons; the bosonic bath describes a fluctuating magnetic field generated by the neighboring local mo-

ments. The locally critical point is identified as a self-consistent solution when the Bose-Fermi Kondo model is treated to the first order in ϵ within an ϵ -expansion renormalization group (RG) procedure. Here $\epsilon \equiv 1 - \gamma$, where γ is the power-law exponent of the spectrum of the dissipative bosonic bath (defined in Eq. (3)).

There are several important questions that remain. First, does a self-consistent locally-critical solution arise at higher orders in ϵ ? Second, what happens to the locally critical point in spin-anisotropic situations? Such anisotropy occurs in heavy fermions since the spin-orbit coupling is usually strong in these systems.

These issues are addressed in the present paper. We carry out a detailed analysis of the Bose-Fermi Kondo model and show that an unstable fixed point exists not only in the spin-isotropic case but also in the presence of spin-anisotropy (both xy and Ising cases). In each case, this critical point describes a continuous transition from a Kondo phase, in which the local moment is quenched by the spins of the conduction electrons, to a “local-moment” phase where there is no Kondo resonance. For the isotropic and xy cases, we calculate the local spin susceptibility to the order ϵ^2 and, in addition, we determine the associated critical exponent, η (defined in Eq. (20)), to all orders in ϵ . In the Ising case, a slightly different RG approach turns out to be useful. [21] In all three cases, we find that the exponent η at the unstable fixed point is

$$\eta = \epsilon \quad (1)$$

This result turns out to guarantee the existence of a self-consistent solution of the locally-critical type in the Kondo lattice systems.

We note in passing that the Bose-Fermi Kondo model is also of interest in other contexts. Historically, the model, in its Ising version, was first studied in the context of an EDMFT treatment of a spinless model in ref. [21], using an ϵ -expansion. The spinful version was subsequently considered, also through an ϵ -expansion,

in refs. [22,23]. The Bose-only Kondo model was later extensively analyzed within a similar ϵ -expansion in ref. [24], which went to higher orders in ϵ and established Eq. (1) for a stable fixed point of that model to all orders. The Bose-only Kondo model appears implicitly in a single-impurity spin model first studied (using a large N approach) in ref. [25]; see also ref. [26]. The Fermi-only Kondo model is of course a standard textbook problem [27].

The remainder of the paper is organized as follows. In Section II, we define the model and introduce the formalism. Section III is devoted to the RG analysis, to order ϵ^2 , in the isotropic case. Section IV presents the explicit calculation of the local spin susceptibility to order ϵ^2 , as well as the analysis of the associated critical exponent η to all orders. In Section V, we consider the effect of spin-anisotropy. Section VI discusses the consequence of our results for the locally critical solution in a Kondo lattice and Section VII provides a brief summary of our results. Some of the technical details are relegated to Appendices A, B C, and D.

II. THE MODEL AND FORMALISM

The Bose-Fermi Kondo model is defined as follows

$$\mathcal{H}_{\text{BFK}} = J \mathbf{S} \cdot \mathbf{s}_c + \sum_{p,\sigma} E_p c_{p\sigma}^\dagger c_{p\sigma} + g \sum_p \mathbf{S} \cdot (\vec{\phi}_p + \vec{\phi}_{-p}^\dagger) + \sum_p w_p \vec{\phi}_p^\dagger \cdot \vec{\phi}_p. \quad (2)$$

A spin- $\frac{1}{2}$ local moment, \mathbf{S} , is coupled to both a fermionic bath ($c_{p\sigma}$), through the Kondo interaction J , and a dissipative vector-bosonic bath ($\vec{\phi}_p$) with a coupling constant g . J is positive, i.e., antiferromagnetic, but g can be either positive or negative. (A sign change in g can be absorbed by a corresponding sign change in ϕ .) The spectral function of the bosonic bath is taken to have a sublinear power-law dependence on energy, at sufficiently low energies:

$$\sum_p [\delta(\omega - w_p) - \delta(\omega + w_p)] = (K_0^2/\pi) |\omega|^\gamma \text{sgn} \omega \quad (3)$$

for $|\omega| < \Lambda$. Here, the power-law exponent

$$0 < \gamma \equiv 1 - \epsilon < 1. \quad (4)$$

The density of states of the conduction electron band near the Fermi energy is taken to be a constant:

$$\sum_p \delta(\omega - E_p) = N_0. \quad (5)$$

The fluctuating magnetic field competes against the Kondo-singlet formation. When the fluctuations are sufficiently slow such that $\epsilon > 0$, g is a relevant coupling in

the RG sense and it can compete with the marginally-relevant J coupling. The physics of this competition is amenable to an ϵ -expansion [22,23,21]. This is in contrast to the Kondo fixed point, which occurs at an infinite coupling and can only be addressed by strong-coupling methods [27].

Following Smith and Si [22,17,18], we adopt the Abrikosov representation of the spin in terms of pseudo- f -electrons [28],

$$\mathbf{S} = \sum_{\sigma\sigma'} f_\sigma^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} f_{\sigma'}, \quad (6)$$

where $\tau^{x,y,z}$ are the Pauli matrices. In this representation, the Bose-Fermi Kondo Hamiltonian takes the following form,

$$\begin{aligned} \tilde{\mathcal{H}} &= \mathcal{H}_0 + \mathcal{H}_J + \mathcal{H}_g, \\ \mathcal{H}_0 &= \sum_\sigma \lambda f_\sigma^\dagger f_\sigma + \sum_{p,\sigma} E_p c_{p\sigma}^\dagger c_{p\sigma} + \sum_p w_p \vec{\phi}_p^\dagger \cdot \vec{\phi}_p, \\ \mathcal{H}_J &= \frac{J}{4} \sum_{\sigma\sigma'} \sigma\sigma' f_\sigma^\dagger f_\sigma c_{\sigma'}^\dagger c_{\sigma'} + \frac{J}{2} (f_\uparrow^\dagger f_\downarrow c_{\downarrow}^\dagger c_{\uparrow} + \text{H.c.}), \\ \mathcal{H}_g &= \frac{g}{2} \sum_\sigma \sigma f_\sigma^\dagger f_\sigma \phi^z + \frac{g}{\sqrt{2}} (f_\uparrow^\dagger f_\downarrow \phi^- + \text{H.c.}), \end{aligned} \quad (7)$$

where λ is an energy level for the f -electron that will be set to ∞ at the end, $\sigma = \pm 1$, $\vec{\phi} \equiv \sum_p (\vec{\phi}_p + \vec{\phi}_{-p}^\dagger)$, and $\phi^\pm = (\phi^x \pm i\phi^y)/\sqrt{2}$.

To analyze the critical behavior of the Bose-Fermi Kondo model, we carry out an RG procedure based on a dimensional regularization with a minimal subtraction (MS) of poles [29,30,31,32]. We define a renormalized field f and a dimensionless coupling constant g by

$$f_B = Z_f^{1/2} f, \quad (8)$$

$$g_B = g Z_f^{-1} Z_g \mu^{\epsilon/2}, \quad (9)$$

where Z_f is the wave-function renormalization factor for f electrons, g_B is the bare coupling constant, Z_g is a coupling constant renormalization of g , and μ is a renormalization energy scale.

The “dimensional” regularization for the conduction electron density of states is done by introducing an ϵ' through

$$\sum_p \delta(\omega - E_p) = N_0 |\omega|^{-\epsilon'}, \quad (10)$$

Accordingly, the dimensionless coupling constant J is defined by

$$J_B = J Z_f^{-1} Z_J \mu^{\epsilon'}. \quad (11)$$

Note that ϵ' is introduced strictly for the purpose of facilitating the MS analysis; it is set to zero at the final stage of the calculation.

III. RG ANALYSIS

In this section, we carry out the RG analysis to order ϵ^2 .

A. RG flow equations

It is known that, to the linear order of ϵ , there is a critical point at $(K_0 g^*)^2 = \epsilon/2$, $(N_0 J^*) = \epsilon/2$ [22,23,17,18]. To obtain the corrections to the order ϵ^2 , we need to calculate the self-energy and vertex corrections beyond the orders J and g^2 and include also terms to the orders g^4 , J^2 , and $g^2 J$. The details of our calculation are given in Appendix A.

The renormalization factors are obtained as

$$\begin{aligned} Z_f &= 1 - \frac{3}{4\epsilon}(K_0 g)^2 - \frac{3}{16\epsilon'}(N_0 J)^2 \\ &\quad - \frac{15}{32\epsilon^2}(K_0 g)^4 + \frac{3}{8\epsilon}(K_0 g)^4 \\ Z_g &= 1 + \frac{1}{4\epsilon}(K_0 g)^2 + \frac{1}{16\epsilon'}(N_0 J)^2 \\ &\quad + \frac{9}{32\epsilon^2}(K_0 g)^4 - \frac{1}{8\epsilon}(K_0 g)^4, \\ Z_J &= 1 - \frac{1}{\epsilon'} N_0 J \\ &\quad + \frac{1}{4\epsilon}(K_0 g)^2 + \frac{1}{16\epsilon'}(N_0 J)^2 + \frac{1}{\epsilon'^2}(N_0 J)^2 \\ &\quad - \left(\frac{1}{\epsilon'(\epsilon + \epsilon')} + \frac{1}{4\epsilon\epsilon'} \right) (K_0 g)^2 (N_0 J) \\ &\quad + \frac{9}{32\epsilon^2}(K_0 g)^4 - \frac{1}{8\epsilon}(K_0 g)^4. \end{aligned} \quad (12)$$

They can then be used to determine the beta functions for the coupling constants g and J , defined as [33]

$$\begin{aligned} \beta(g) &= \mu \frac{dg}{d\mu} \Big|_{g_B, J_B}, \\ \beta(J) &= \mu \frac{dJ}{d\mu} \Big|_{g_B, J_B}. \end{aligned} \quad (13)$$

The results are given as follows

$$\begin{aligned} \beta(g) &= -g \left(\frac{\epsilon}{2} - (K_0 g)^2 + (K_0 g)^4 - \frac{(N_0 J)^2}{2} \right), \\ \beta(J) &= -J \left((N_0 J) - \frac{(N_0 J)^2}{2} \right) \\ &\quad - J \left(-(K_0 g)^2 + (K_0 g)^4 \right). \end{aligned} \quad (14)$$

B. The phase diagram

The RG equations (14) yield several fixed points. There are two stable ones. The fixed point located at

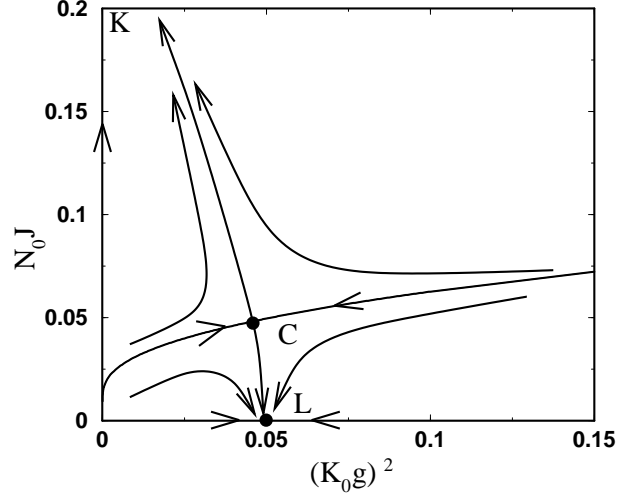


FIG. 1. RG flows in $g - J$ plane when $\epsilon = 0.1$. K (Kondo) and L (local-moment) denote the two stable fixed points. The separatrix is determined numerically. C denotes the unstable fixed point (the critical point).

$g^* = 0$ and large J^* (“K” in Fig. 1), as usual, specifies the Kondo phase [34]. Here, the local moment is quenched by the spins of the conduction electrons, leading to the development of a Kondo resonance.

Another stable fixed point (“L” in Fig. 1) is located at

$$\begin{aligned} (K_0 g^*)^2 &= \frac{\epsilon}{2} + \frac{1}{4}\epsilon^2 + O(\epsilon^3) \\ (N_0 J^*) &= 0 \end{aligned} \quad (15)$$

We will call it a “local-moment” fixed point, emphasizing the fact that it describes a phase in which no Kondo resonance arises. The dynamics in the local-moment phase is controlled by the coupling of the local moment to the dissipative bosonic bath alone.

A separatrix specifies the boundary of the domains of attractions for these two phases in the J - g parameter space. Lying on this separatrix is an unstable fixed point, or a critical point (“C” in Fig. 1). It is located at

$$\begin{aligned} (K_0 g^*)^2 &= \frac{\epsilon}{2} + \frac{1}{8}\epsilon^2 + O(\epsilon^3) \\ (N_0 J^*) &= \frac{\epsilon}{2} + O(\epsilon^3) \end{aligned} \quad (16)$$

The critical point marks the point where the spectral weight of the Kondo resonance goes to zero. It captures the competition between the Kondo coupling of the local moment to the conduction electrons on the one hand, and its coupling to the fluctuating magnetic field on the other.

The RG flow can be solved numerically. In Fig. 1, we give the result for $\epsilon = 0.1$.

IV. THE LOCAL DYNAMICAL SPIN SUSCEPTIBILITY

We now turn to the calculation of the local susceptibility.

A. Critical susceptibility to order ϵ^2

Consider the local spin susceptibility to order ϵ^2 . We first calculate the bare spin susceptibility, i.e., the autocorrelation function of the unrenormalized local spin operator specified by Eq. (6),

$$\begin{aligned}\chi_B(\tau) &\equiv \frac{1}{2} \langle T_\tau S^-(\tau) S^+(0) \rangle = \lim_{\lambda \rightarrow \infty} \frac{1}{2} e^{\beta\lambda} \tilde{\chi}_B(\tau), \\ \tilde{\chi}_B(\tau) &\equiv \frac{1}{2} \langle T_\tau f_\downarrow^\dagger(\tau) f_\uparrow(\tau) f_\uparrow^\dagger(0) f_\downarrow(0) \rangle,\end{aligned}\quad (17)$$

where $S^\pm \equiv (S^x \pm iS^y)$. (For notational simplicity, we've dropped the subscript B in the operators used in Eq. (17).)

Correspondingly, we define a spin renormalization factor Z which relates the renormalized spin susceptibility, χ , to the bare susceptibility,

$$\chi(\tau) = \frac{1}{Z} \chi_B(\tau), \quad (18)$$

The calculational details are given in Appendix B. From Eq. (B6), Z is given by

$$Z = 1 - \frac{2(K_0g)^2}{\epsilon} + \frac{(K_0g)^4}{\epsilon} - \frac{(N_0J)^2}{2\epsilon'}. \quad (19)$$

At the critical point, the local spin susceptibility is expected to have a power-law form:

$$\chi(\tau) \approx A_\epsilon \left(\frac{\tau_0}{|\tau|} \right)^\eta, \quad (20)$$

for $|\tau| \gg \tau_0$, where $\tau_0 = 1/\Lambda$ is a cut-off scale. Here, η is the anomalous dimension. It can be calculated from the renormalization factor Z ,

$$\begin{aligned}\eta &= \mu \frac{d \ln Z}{d \mu} \Big|_{g=g^*, J=J^*} \\ &= \left(\beta(g) \frac{\partial \ln Z}{\partial g} + \beta(J) \frac{\partial \ln Z}{\partial J} \right) \Big|_{g=g^*, J=J^*} \\ &= 2(K_0g^*)^2 - 2(K_0g^*)^4 + (N_0J^*)^2 \\ &= \epsilon.\end{aligned}\quad (21)$$

It is striking that, although the order ϵ^2 corrections are present both in the beta functions and in the critical coupling constants, they cancel out in the critical exponent η , leaving only a non-zero linear term.

The critical amplitude is given in Appendix B.

For completeness, we also briefly discuss the local spin susceptibility at the local-moment fixed point. We assume that the susceptibility here also has a power-law form:

$$\chi(\tau) \sim A_\epsilon^L \left(\frac{\tau_0}{|\tau|} \right)^{\eta_L}, \quad (22)$$

with an anomalous dimension η_L . η_L has the same dependence on g^*, J^* as in Eq. (21). Even though the coupling constants g^*, J^* now take the values specified by Eq. (15), it turns out that the ϵ^2 terms again cancel with each other leaving

$$\eta_L = \epsilon. \quad (23)$$

The critical amplitude is also given in Appendix B.

B. Critical exponent to all orders in ϵ

In this section, we show that $\eta = \epsilon$ given in Eq. (21) is in fact exact to all orders.

First, we explore the reason of the cancellation of the ϵ^2 terms in η seen in Eq. (21). Using the equations (12), (19), we can easily verify that

$$Z^{-1} = (Z_f^{-1} Z_g)^2 \quad (24)$$

both to linear and second orders in ϵ . Combining Eq. (24) with Eq. (9) lead to

$$g_B = g Z^{-1/2} \mu^{\epsilon/2}. \quad (25)$$

Differentiating the logarithm of both sides with respect to $\ln \mu$, keeping g_B and J_B fixed, we have

$$\eta = \epsilon + \frac{2\beta(g)}{g} \Big|_{g=g^*, J=J^*}. \quad (26)$$

At the fixed point, $\beta(g) = 0$, which gives

$$\eta = \epsilon. \quad (27)$$

There is also a consistency check for our result. We can start from Eq. (11). Combining this equation with Eq. (24) lead to

$$J_B = J Z^{-1/2} (Z_J/Z_g). \quad (28)$$

(We have set $\epsilon' = 0$.) Again, differentiating the logarithm of both sides with respect to $\ln \mu$, and again keeping g_B and J_B fixed, we end up with

$$\eta = 2 \frac{d \ln (Z_J/Z_g)}{d \ln \mu} \Big|_{g=g^*, J=J^*}. \quad (29)$$

To be consistent, the RHS should be equal to ϵ . This is indeed verified to both linear and quadratic orders in ϵ .

We now proceed to higher order contributions. We show in Appendix B that Eq. (24) is valid to all orders. Eq. (26) is then valid to all orders in ϵ . By extension, η equals to ϵ to all orders in ϵ . Our reasoning basically parallels that of ref. [24] for the stable fixed point in the Bose-only Kondo problem.

V. ANISOTROPIC CASES (XY AND ISING)

So far, we have discussed the spin-isotropic case. We now turn to the effect of anisotropy in spin-space.

A. RG equations

We introduce separate parameters for the transverse and longitudinal spin couplings as follows:

$$\begin{aligned}\mathcal{H}_J &= \frac{J_z}{4} \sum_{\sigma\sigma'} \sigma\sigma' f_\sigma^\dagger f_\sigma c_{\sigma'}^\dagger c_{\sigma'} + \frac{J_\perp}{2} (f_\uparrow^\dagger f_\downarrow c_\uparrow^\dagger c_\downarrow + \text{H.c.}), \\ \mathcal{H}_g &= \frac{g_z}{2} \sum_{\sigma} \sigma f_\sigma^\dagger f_\sigma \phi^z + \frac{g_\perp}{\sqrt{2}} (f_\uparrow^\dagger f_\downarrow \phi^- + \text{H.c.}),\end{aligned}\quad (30)$$

The RG procedure parallels that for the isotropic case, except that we need to differentiate between the longitudinal and transverse couplings in all the contributions, and associate different renormalization factors for the different types of vertices:

$$\begin{aligned}g_i^B &= g_i Z_f^{-1} Z_{g_i} \mu^{\epsilon/2} \\ J_i^B &= J_i Z_f^{-1} Z_{J_i} \mu^{\epsilon'},\end{aligned}\quad (31)$$

where $i = \perp, z$. Up to the order of our interest, the results for the renormalization factors are given in Appendix C. The resulting RG equations are given as follows,

$$\begin{aligned}\beta(g_\perp) &= -\frac{\epsilon}{2} g_\perp + \frac{(K_0 g_\perp)^2 + (K_0 g_z)^2}{2} g_\perp \\ &\quad - \frac{(K_0 g_\perp)^2 (K_0 g_z)^2 + (K_0 g_\perp)^4}{2} g_\perp \\ &\quad + \frac{(N_0 J_\perp)^2 + (N_0 J_z)^2}{4} g_\perp\end{aligned}\quad (32)$$

$$\begin{aligned}\beta(g_z) &= -\frac{\epsilon}{2} g_z + (K_0 g_\perp)^2 g_z \\ &\quad - (K_0 g_\perp)^2 (K_0 g_z)^2 g_z \\ &\quad + \frac{(N_0 J_\perp)^2}{2} g_z\end{aligned}\quad (33)$$

$$\begin{aligned}\beta(J_\perp) &= \frac{(K_0 g_\perp)^2 + (K_0 g_z)^2}{2} J_\perp \\ &\quad - \frac{(K_0 g_\perp)^2 (K_0 g_z)^2 + (K_0 g_\perp)^4}{2} J_\perp \\ &\quad - (N_0 J_z) J_\perp + \frac{(N_0 J_\perp)^2 + (N_0 J_z)^2}{4} J_\perp\end{aligned}\quad (34)$$

$$\begin{aligned}\beta(J_z) &= (K_0 g_\perp)^2 J_z \\ &\quad - (K_0 g_\perp)^2 (K_0 g_z)^2 J_z \\ &\quad - (N_0 J_\perp) J_z + \frac{(N_0 J_\perp)^2}{2} J_z\end{aligned}\quad (35)$$

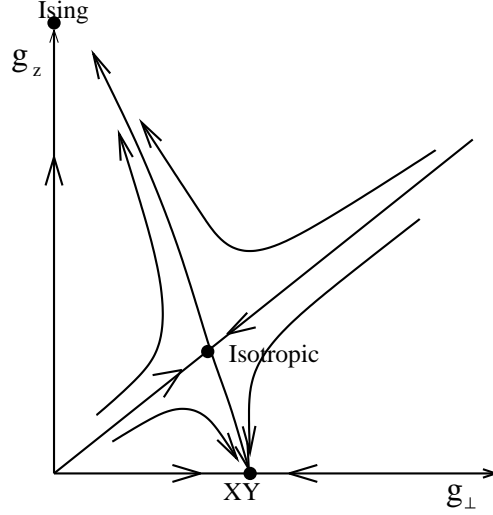


FIG. 2. Schematic RG flow diagram for anisotropic g coupling when $J = 0$. The Ising fixed-point and the xy fixed-point are stable while the isotropic fixed point is unstable.

Consider first the $J = 0$ case, i.e. the Bose-only Kondo model. The RG flow is given in Fig. 2. There are two more fixed points in addition to the isotropic one. The xy fixed point is located at

$$\begin{aligned}(K_0 g_\perp^*)^2 &= \epsilon + \epsilon^2 + O(\epsilon^3) \\ (K_0 g_z^*) &= 0.\end{aligned}\quad (36)$$

While the Ising fixed point is nominally located at $g_\perp^* = 0$ and $g_z^* = \infty$. Both the isotropic fixed point and the xy fixed point are accessible by the ϵ -expansion. The Ising fixed point, on the other hand, is beyond the reach of the perturbative RG scheme.

The J -coupling can lead away from the “local-moment” fixed point not only in the isotropic case, but also in the xy and Ising cases. The isotropic case has already been discussed in the previous sections. We now turn to the xy and Ising cases, respectively.

B. Critical behavior in the xy case

We will now set $g_z = 0$ in Eqs. (32)- (35). [Eq. (33) implies that g_z will stay at zero under the RG transformation when its initial value vanishes.] An unstable fixed point occurs at

$$\begin{aligned}(K_0 g_\perp^*)^2 &= \epsilon + \frac{5\epsilon^2}{8} + O(\epsilon^3) \\ N_0 J_\perp^* &= \frac{\epsilon}{\sqrt{2}} + \frac{7}{16\sqrt{2}} \epsilon^2 + O(\epsilon^3) \\ N_0 J_z^* &= \frac{\epsilon}{2} + O(\epsilon^3)\end{aligned}\quad (37)$$

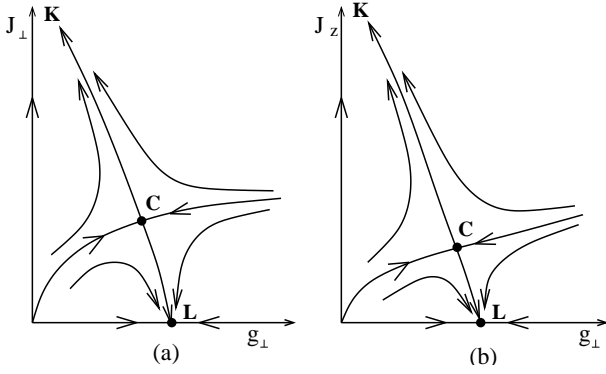


FIG. 3. Schematic RG flow diagram for the xy case. Here we show the projection of the RG flow in the three dimensional parameter space (J_\perp, J_z, g_\perp) onto a) the $J_\perp - g_\perp$ plane and b) the $J_z - g_\perp$ plane.

Note that g_z remains irrelevant near this fixed point, establishing the consistency of our analysis. The RG flows are shown in Fig. 3.

The perturbative correction to the local spin susceptibility can be calculated as in the isotropic case. The most general expression for the susceptibility as well as its renormalization factor Z are given in Appendix C. The corresponding expression for η is also given there, in Eq. (C9). Using the values of the coupling constants at this fixed point, Eq. (37), we find that η remains to be ϵ to order ϵ^2 .

The same argument made in Appendix B for the isotropic case carries through here for the xy-case as well, resulting in

$$Z^{-1} = (Z_f^{-1} Z_{g_\perp})^2, \quad (38)$$

to infinite orders in ϵ . So $\eta = \epsilon$ is again valid to all orders in ϵ .

We now again briefly examine the local spin susceptibility at the local-moment fixed point in this case, where $J_\perp^* = J_z^* = 0$, and

$$(K_0 g_\perp^*)^2 = \epsilon + \epsilon^2 + O(\epsilon^3). \quad (39)$$

The anomalous dimension η_L has the same expression as in Eq. (C12). Substituting the values of the stable fixed point, we find that η_L is equal to ϵ , just as in the isotropic case.

C. Critical behavior in the Ising case

None of the non-trivial fixed points in the Ising case is within the reach of the perturbative RG scheme. For the unstable fixed point, for instance, setting $g_\perp = 0$ in Eqs. (32-35) will yield a $(N_0 J_\perp^*)^2 \sim \epsilon$ that is still small, but J_z^* and g_z^* that are of order unity. The latter violates our starting assumption.

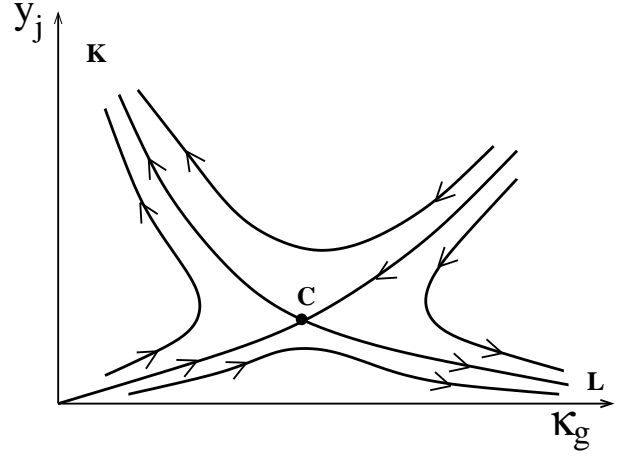


FIG. 4. Schematic RG flow diagram for the Ising case, projected onto the plane spanned by $y_j - \kappa_g$ plane.

This problem, however, has already been studied in ref. [21]. A finite J_z and g_z can be handled by introducing a so-called “kink-gas” representation. In this representation, the unstable fixed point is still accessible by an ϵ -expansion. The calculation is outlined in Appendix D. The RG equations are no longer constructed in terms of the bare couplings. Instead, they are given in terms of the stiffness constants κ_j, κ_g , and the fugacity y_j , whose initial values are specified by the bare parameters as follows,

$$\begin{aligned} \kappa_j^B &= [1 - \frac{1}{\pi} \tan^{-1}(\frac{\pi}{4} N_0 J_z^B)]^2 \\ \kappa_g^B &= \frac{\Gamma(\gamma)}{4} \tau_0^{1-\gamma} (K_0 g_z^B)^2 \\ y_j^B &= \frac{N_0 J_\perp^B}{2} \end{aligned} \quad (40)$$

The RG flow projected onto the $y_j - \kappa_g$ plane is shown in Fig. 4. In the vicinity of the unstable fixed point, κ_j is irrelevant.

The exponent η at the unstable fixed point is calculated in Appendix D to the linear order in ϵ . Again, it is found that

$$\eta = \epsilon \quad (41)$$

In the same Appendix, it is also shown that η is in fact the same as that for the critical point of a classical ferromagnetic Ising chain with a long-range interaction that decays in distance in terms of a power-law exponent $2 - \epsilon$. For this problem, it has long been held that Eq. (41) is in fact exact [35,36].

In passing, we note that this analogy also allows us to state what happens to the local susceptibility at the local-moment fixed point shown in Fig. (4). Here, $\chi(\tau)$ picks up a constant (τ -independent) piece. At the same time, the “connected” susceptibility (defined such that

the constant piece does not appear) decays with an exponent $\eta_L = 2 - \epsilon$ [37].

VI. LOCALLY CRITICAL POINT OF THE KONDO LATTICE

We now discuss the consequences of our results for the locally critical point of the Kondo lattices. A self-consistent microscopic treatment of the Kondo lattice model has been presented in detail elsewhere [17,18]. This was carried out within the EDMFT approach developed in [21,38,39]. (This approach is a generalization of the dynamical mean field theory [40] such that magnetic fluctuations are also taken into account.) The self-consistent equation reads as follows,

$$\langle \chi(\mathbf{q}, \omega) \rangle_{\mathbf{q}} = \chi_{\text{loc}}(\omega), \quad (42)$$

Here, the lattice dynamical spin susceptibility has the form

$$\chi(\mathbf{q}, \omega) = \frac{1}{M(\omega) + I_{\mathbf{q}}}, \quad (43)$$

where $M(\omega)$ is the “spin self-energy” and $I_{\mathbf{q}}$ describes the exchange (RKKY) interaction between the local moments. Eq. (42) is simply a statement of translational invariance, as it equates the on-site spin susceptibility of the lattice system (LHS) to the susceptibility of any local moment (RHS). The locally critical solution arises when $\chi_{\text{loc}}(\omega)$ is singular – signaling the destruction of the Kondo resonance as discussed in previous sections – at the point where the peak susceptibility $\chi(\mathbf{Q}_{AF}, \omega)$ just diverges. It is shown in refs. [17,18] that one requirement for a locally critical point is that the magnetic fluctuations are two-dimensional. This can be easily seen from Eqs. (42,43). The fact that the peak susceptibility $\chi(\mathbf{Q}_{AF}, \omega)$ diverges means that $M(0) = -I_{\mathbf{Q}_{AF}}$. Recognizing that for $\mathbf{q} \sim \mathbf{Q}_{AF}$, the interaction has the generic form, $I_{\mathbf{q}} = I_{\mathbf{Q}_{AF}} + a(\mathbf{q} - \mathbf{Q}_{AF})^2$, we have

$$\chi(\mathbf{q} \sim \mathbf{Q}_{AF}, \omega \rightarrow 0) = \frac{1}{a(\mathbf{q} - \mathbf{Q}_{AF})^2} \quad (44)$$

Now, the fact that the local susceptibility, $\chi_{\text{loc}}(\omega \rightarrow 0)$, is singular demands that the \mathbf{q} -averaging of the LHS of Eq. (42) should diverge. In other words, integrating the RHS of Eq. (44) over \mathbf{q} should yield a singular result. Two-dimensionality provides the phase space for this.

This condition, however, is not sufficient. It turns out that a second condition must be met for the existence of such a locally-critical solution. The condition is precisely Eq. (1). The origin of this second condition is somewhat subtle, but has to do with the fact that $M(\omega)$ and χ_{loc} , in addition to being related through Eqs. (42,43), must also satisfy a Dyson-like equation of the (self-consistent) Bose-Fermi Kondo model. The details can be found in ref. [18].

By proving that Eq. (1) is valid to all orders in ϵ , and also in spin-anisotropic cases, we have then established that the locally critical point is indeed a robust result within these microscopic considerations. (The arguments for the robustness of the locally critical point beyond the microscopic approaches can be found in refs. [17,18].)

We stress that a key element of the locally critical point of the Kondo lattice is that, the criticality of the local degrees of freedom is embedded in the criticality associated with the long-range ordering in the lattice. In other words, the point where a local energy scale turns to zero coincides with where the spatial correlation length just diverges. This is very different from the self-consistent spin liquid solutions discussed in other contexts [25,26], which correspond to a phase instead of a critical point. This difference is also reflected in the fact that stabilizing the locally critical solution in Kondo lattices requires two-dimensional magnetic fluctuations.

VII. SUMMARY

In short, we have carried out a detailed analysis of the Bose-Fermi Kondo model both when the spin-rotational invariance is satisfied (isotropic) and when it is broken (xy and Ising). In each case, we have identified a critical point that separates a Kondo phase, where the local moment is quenched by the spins of the conduction electrons, and a local moment phase where there exists no Kondo resonance. This unstable fixed point marks the point where the spectral weight associated with the Kondo resonance has just gone down to zero.

In all three cases, we find that the exponent for the local spin susceptibility at the unstable fixed point is equal to ϵ . We note that the three cases correspond to three different one-dimensional statistical mechanical problems. It is remarkable that, the susceptibility exponent at the unstable fixed point is insensitive to these differences. However, the same cannot be said about the stable fixed points: here, the susceptibility exponent for the Ising case is very different from the isotropic and xy cases.

Our results have important consequences for the locally critical behavior in Kondo lattices. In particular, it guarantees that a quantum phase transition of a locally-critical type [17,18,19] is a robust microscopic solution to the Kondo lattices: it arises to all orders in ϵ and, in addition, not only when the system is spin-rotationally invariant but also in spin-anisotropic situations.

ACKNOWLEDGMENTS

This work has been supported in part by TCSUH, the Robert A. Welch Foundation, and NSF Grant No. DMR-0090071. We would also like to acknowledge the

support of Argonne National Laboratory, the University of Chicago, and the University of Illinois at Urbana-Champaign during our leave of absence from Rice University. This work was briefly reported in ref. [42]. We have recently learned [43] that G. Zarand and E. Demler have also analyzed this problem, with results which are similar to ours.

APPENDIX A: RG PROCEDURE TO ORDER ϵ^2

In this appendix, we give the details of the RG analysis. We adopt the dimensional regularization and minimal subtraction scheme [29,30,31,32].

We first carry out a renormalized perturbation calculation. The quantities of interest are the J - and g -vertices:

$$\begin{aligned}\Gamma_J(\omega + \lambda) &\equiv J\mu^{\epsilon'}\gamma_J(\omega + \lambda) \\ \Gamma_g(\omega + \lambda) &\equiv g\mu^{\epsilon/2}\gamma_g(\omega + \lambda)\end{aligned}\quad (\text{A1})$$

as well as the f -electron self-energy, $\Sigma_f(\omega + \lambda)$.

$$G^{-1}(\omega + \lambda) = \omega - \Sigma_f(\omega + \lambda) = Z_f G_B^{-1}(\omega + \lambda) \quad (\text{A2})$$

Within the renormalized perturbation calculation, the bare parameters in Eq. (7) are replaced with the corresponding renormalized ones. We end up with a Hamiltonian containing one part that has the form of Eq. (2) (the tree level Hamiltonian) and another part which describes the counterterms:

$$\begin{aligned}\mathcal{H}_{ct} = & \sum_{\sigma} (Z_f - 1)\lambda f_{\sigma}^{\dagger} f_{\sigma} + (Z_J - 1)J\mu^{\epsilon'} \mathbf{S} \cdot \mathbf{s}_c \\ & + (Z_g - 1)g\mu^{\epsilon/2} \mathbf{S} \cdot \vec{\phi},\end{aligned}\quad (\text{A3})$$

There are then two types of perturbative diagrams, one coming from the tree level Hamiltonian, which we will call direct perturbative contributions below, and the other from the counterterms.

The singular contributions are kept track of through poles as a function of ϵ and ϵ' . By demanding that such poles are “minimally” removed so that the renormalized quantities Γ_g , Γ_J , and G_f are regular (as a function of ϵ and ϵ'), we fix the wavefunction and vertex renormalization factors Z_f , Z_J , and Z_g order by order in the coupling constants. From these renormalization factors, we can determine the RG beta functions as well as the anomalous dimensions. Note that what we are carrying through is a double expansion in ϵ and ϵ' .

In the following, we separate our discussions according to the order in J and g .

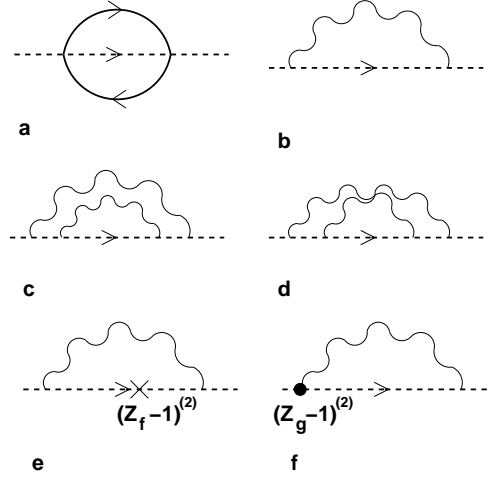


FIG. 5. Self-energy diagrams for the f -electrons. The dashed, solid and wavy lines denote the propagators of the f -electrons, conduction electrons and vector bosons, respectively. The cross in (e) and dot in (f) denote the counterterm $Z_f - 1$ and $Z_g - 1$, respectively.

1. First order result

To the first order of perturbation, there are only corrections to the J -coupling from processes that involve the J -coupling alone. The diagrams are specified by Figs. 5(a)(b) in Ref. [18]. The result is given by

$$\begin{aligned}\gamma_J^{(1)}(\omega + \lambda) &= \frac{N_0 J}{\epsilon'} \left(\frac{\mu}{-\omega} \right)^{\epsilon'} \\ &= \frac{N_0 J}{\epsilon'} + N_0 J \ln \left(\frac{\mu}{-\omega} \right) + \dots\end{aligned}\quad (\text{A4})$$

Adding the counterterm $(Z_J - 1)$ and demanding that the pole terms cancel, we find that

$$(Z_J - 1)^{(1)} = -\frac{1}{\epsilon'} (N_0 J). \quad (\text{A5})$$

2. Second order result

To the order g^2 and J^2 , the contributions to the f -electron self-energy are shown in Figs. 5a and 5b and they yield the following expression:

$$\begin{aligned}\Sigma_f^{(2)}(\omega + \lambda) &= -\omega \frac{3}{4\epsilon} (K_0 g)^2 \left(\frac{\mu}{-\omega} \right)^{\epsilon} \\ &\quad -\omega \frac{3}{16\epsilon'} (N_0 J)^2 \left(\frac{\mu}{-\omega} \right)^{2\epsilon'},\end{aligned}\quad (\text{A6})$$

By demanding that these poles of the self-energy be cancelled by the counterterm contribution $-(Z_f - 1)\omega$, we fix

$$(Z_f - 1)^{(2)} = -\frac{3}{4\epsilon}(K_0 g)^2 - \frac{3}{16\epsilon'}(N_0 J)^2. \quad (\text{A7})$$

To the same order, the corrections to the coupling constant g are specified by Fig. 6 of Ref. [18] and they yield

$$\begin{aligned} \gamma_g^{(2)}(\omega + \lambda) &= -\frac{1}{4\epsilon}(1 - \epsilon)(K_0 g)^2 \left(\frac{\mu}{-\omega}\right)^\epsilon \\ &\quad - \frac{1}{16\epsilon'}(N_0 J)^2 \left(\frac{\mu}{-\omega}\right)^{2\epsilon'}, \end{aligned} \quad (\text{A8})$$

This fixes the second-order contribution to Z_g ,

$$(Z_g - 1)^{(2)} = \frac{1}{4\epsilon}(K_0 g)^2 + \frac{1}{16\epsilon'}(N_0 J)^2. \quad (\text{A9})$$

The corrections to the coupling constant J include not only the direct perturbation diagrams, but also contributions from the first order counterterm $(Z_J - 1)^{(1)}J$. Figs. 5(c)(d) in Ref. [18] give

$$\begin{aligned} \gamma_J^{(2)I}(\omega + \lambda) &= -\frac{1}{4\epsilon}(1 - \epsilon)(K_0 g)^2 \left(\frac{\mu}{-\omega}\right)^\epsilon \\ &\quad - \frac{1}{16\epsilon'}(N_0 J)^2 \left(\frac{\mu}{-\omega}\right)^{2\epsilon'}. \end{aligned} \quad (\text{A10})$$

The so-called “parquet” diagrams as in Kondo problems, shown in Figs. 6(a)-(f), give

$$\gamma_J^{(2)II}(\omega + \lambda) = \frac{(N_0 J)^2}{\epsilon'^2} \left(\frac{\mu}{-\omega}\right)^{2\epsilon'}. \quad (\text{A11})$$

The diagrams with the counterterm $(Z_J - 1)^{(1)}J$, shown in Figs. 6(g)(h), give

$$\begin{aligned} \gamma_J^{(2)III}(\omega + \lambda) &= 2(Z_J - 1)^{(1)}\gamma_J^{(1)}(\omega + \lambda) \\ &= -\frac{2(N_0 J)^2}{\epsilon'^2} \left(\frac{\mu}{-\omega}\right)^{\epsilon'}, \end{aligned} \quad (\text{A12})$$

where the prefactor 2 is a symmetry factor. From these contributions, we obtain the second order correction to Z_J :

$$(Z_J - 1)^{(2)} = \frac{1}{4\epsilon}(K_0 g)^2 + \frac{1}{16\epsilon'}(N_0 J)^2 + \frac{1}{\epsilon'^2}(N_0 J)^2. \quad (\text{A13})$$

3. Third order result

To the third order of perturbation, we need only to consider the $g^2 J$ order corrections to the J vertices. (There are also J^3 order corrections to the J vertices, but these contributions are higher than ϵ^2 order.) The direct perturbative diagrams are shown in Fig. 7. Summing up the contributions of these diagrams, we get

$$\begin{aligned} \gamma_J^{(3)I}(\omega + \lambda) &= \left(\frac{1}{4\epsilon'} - \frac{1}{4\epsilon\epsilon'} - \frac{1}{\epsilon(\epsilon + \epsilon')} \right) \\ &\quad \times (K_0 g)^2 (N_0 J) \left(\frac{\mu}{-\omega}\right)^{\epsilon + \epsilon'}. \end{aligned} \quad (\text{A14})$$

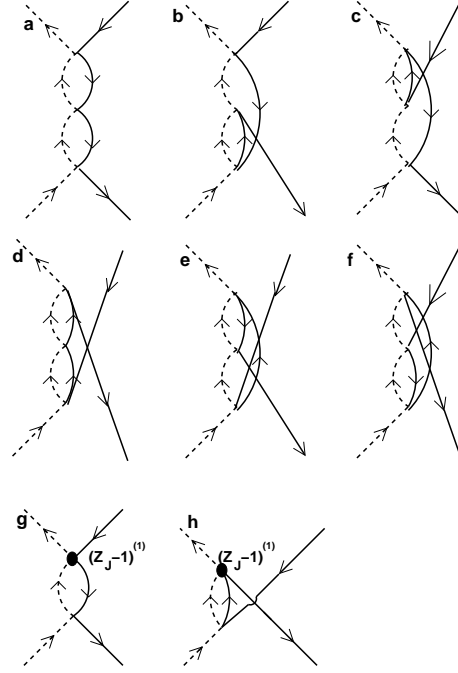


FIG. 6. The vertex correction diagrams for the coupling J to the J^2 order. Here, (a)-(f) are the “parquet” diagrams; (g)(h) are diagrams with counterterms $(Z_J - 1)^{(1)}$.

Fig. 8 shows the diagrams with counterterms to $g^2 J$ order. The contributions from Figs. 8(a)(b) give rise to

$$\begin{aligned} \gamma_J^{(3)II}(\omega + \lambda) &= -(Z_f - 1)^{(2)}\gamma_J^{(1)}(\omega + \lambda) \\ &= \frac{3(K_0 g)^2 (N_0 J)}{4\epsilon\epsilon'} \left(\frac{\mu}{-\omega}\right)^{\epsilon'}; \end{aligned} \quad (\text{A15})$$

those from Figs. 8(c)(d) yield

$$\begin{aligned} \gamma_J^{(3)III}(\omega + \lambda) &= 2(Z_J - 1)^{(2)}\gamma_J^{(1)}(\omega + \lambda) \\ &= \frac{(K_0 g)^2 (N_0 J)}{2\epsilon\epsilon'} \left(\frac{\mu}{-\omega}\right)^{\epsilon'}; \end{aligned} \quad (\text{A16})$$

and finally, Fig. 8(e) leads to

$$\begin{aligned} \gamma_J^{(3)IV}(\omega + \lambda) &= (Z_J - 1)^{(1)}\gamma_J^{(2)}(\omega + \lambda) \\ &= \left(\frac{1}{4\epsilon\epsilon'} - \frac{1}{4\epsilon'} \right) (K_0 g)^2 (N_0 J) \left(\frac{\mu}{-\omega}\right)^\epsilon. \end{aligned} \quad (\text{A17})$$

Summing up $\gamma_J^{(3)I,II,III,IV}$ and subtracting poles, we have

$$(Z_J - 1)^{(3)} = - \left(\frac{1}{\epsilon'(\epsilon + \epsilon')} + \frac{1}{4\epsilon\epsilon'} \right) (K_0 g)^2 (N_0 J). \quad (\text{A18})$$

Note that the simple poles in Eqs. (A14,A17) cancel with each other, leaving only double poles in the net result, Eq. (A18). Therefore, no $g^2 J$ terms will appear in the beta functions.

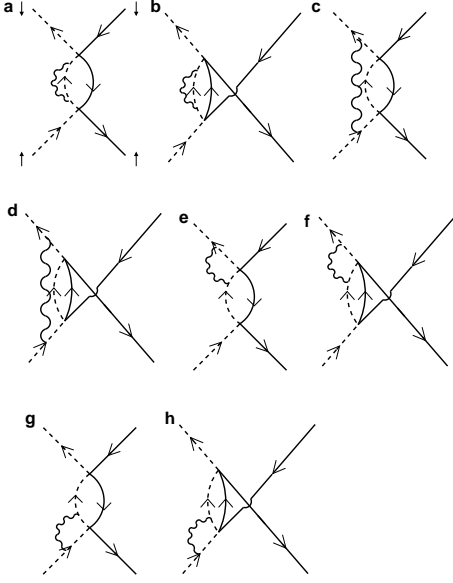


FIG. 7. The direct perturbative vertex diagrams for the coupling J to the $g^2 J$ order.

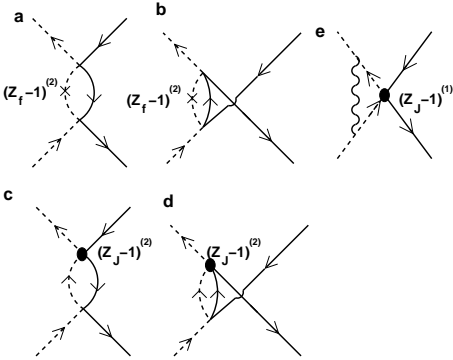


FIG. 8. The vertex correction diagrams for the coupling J to the $g^2 J$ order with counterterms $(Z_f - 1)^{(2)}$, $(Z_J - 1)^{(1)}$ and $(Z_J - 1)^{(2)}$.

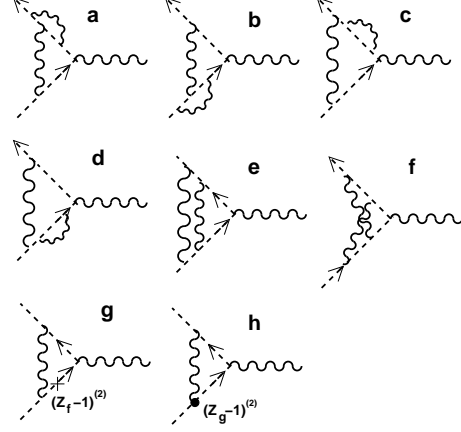


FIG. 9. The vertex correction diagrams for the coupling g to the g^4 order. (a)-(f) are the direct perturbative diagrams while (g), (h) are diagrams with counterterms

4. Fourth order result

We now turn to contributions of order g^4 , the only terms we are interested in to this order. First, we consider the f -electron self-energy. Figs. 5(c), (d) are the direct perturbative contributions to f -electron self-energy of this order. Figs. 5(e), (f), on the other hand, are the diagrams with counter terms of order g^2 and J^2 . Keeping only the g^4 order terms, we have:

$$\begin{aligned}\Sigma_f^{(4)(5c+5d)}(\omega + \lambda) &= \omega \left(\frac{15}{32\epsilon^2} + \frac{3}{8\epsilon} \right) (K_0 g)^4 \left(\frac{\mu}{-\omega} \right)^{2\epsilon}, \\ \Sigma_f^{(4)(5e)}(\omega + \lambda) &= -(Z_f - 1)^{(2)} \Sigma_f^{(2)}(\omega + \lambda) \\ &= -\omega \frac{9}{16\epsilon^2} (K_0 g)^4 \left(\frac{\mu}{-\omega} \right)^\epsilon \\ \Sigma_f^{(4)(5f)}(\omega + \lambda) &= 2(Z_g - 1)^{(2)} \Sigma_f^{(2)}(\omega + \lambda) \\ &= -\omega \frac{3}{8\epsilon^2} (K_0 g)^4 \left(\frac{\mu}{-\omega} \right)^\epsilon\end{aligned}\quad (\text{A19})$$

The resulting corrections to Z_f in this order is given by

$$(Z_f - 1)^{(4)} = -\frac{15}{32\epsilon^2} (K_0 g)^4 + \frac{3}{8\epsilon} (K_0 g)^4. \quad (\text{A20})$$

Next, we consider the g -vertex corrections. They are specified in Fig. 9. The contributions from the direct perturbative diagrams, Figs. 9(a)-(f), sum up to

$$\gamma_g^{(4) a-f}(\omega + \lambda) = \left(\frac{9}{32\epsilon^2} - \frac{7}{16\epsilon} \right) (K_0 g)^4 \left(\frac{\mu}{-\omega} \right)^{2\epsilon}; \quad (\text{A21})$$

while those from the counterterms, Figs. 9(g)(h), yield,

$$\begin{aligned}\gamma_g^{(4) g,h}(\omega + \lambda) &= 3(Z_g - 1)^{(2)} \gamma_g^{(2)}(\omega + \lambda) \\ &\quad - 2(Z_f - 1)^{(2)} \gamma_g^{(2)}(\omega + \lambda) \\ &= \left(-\frac{9}{16\epsilon^2} + \frac{9}{16\epsilon} \right) (K_0 g)^4 \left(\frac{\mu}{-\omega} \right)^\epsilon.\end{aligned}\quad (\text{A22})$$

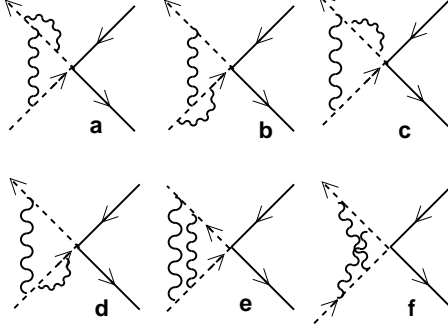


FIG. 10. The vertex correction diagrams for the coupling J to the g^4 order.

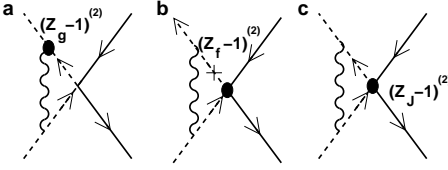


FIG. 11. The vertex correction diagrams for the coupling J to the g^4 order with counterterms.

Therefore, the correction to Z_g is given by

$$(Z_g - 1)^{(4)} = \frac{9}{32\epsilon^2}(K_0g)^4 - \frac{1}{8\epsilon}(K_0g)^4. \quad (\text{A23})$$

Similarly, the J -coupling corrections contain both direct perturbative diagrams (Fig. 10) and diagrams with counterterms (Fig. 11) which are given by

$$\gamma_J^{(4)I}(\omega + \lambda) = \left(\frac{9}{32\epsilon^2} - \frac{7}{16\epsilon} \right) (K_0g)^4 \left(\frac{\mu}{-\omega} \right)^{2\epsilon}, \quad (\text{A24})$$

and

$$\begin{aligned} \gamma_J^{(4)II}(\omega + \lambda) &= \left(2(Z_g - 1)^{(2)} - 2(Z_f - 1)^{(2)} \right) \gamma_J^{(2)}(\omega + \lambda) \\ &\quad + (Z_J - 1)^{(2)} \gamma_J^{(2)}(\omega + \lambda) \\ &= \left(-\frac{9}{16\epsilon^2} + \frac{9}{16\epsilon} \right) (K_0g)^4 \left(\frac{\mu}{-\omega} \right)^\epsilon, \end{aligned} \quad (\text{A25})$$

respectively. The resulting correction to Z_J is

$$(Z_J - 1)^{(4)} = \frac{9}{32\epsilon^2}(K_0g)^4 - \frac{1}{8\epsilon}(K_0g)^4. \quad (\text{A26})$$

We can now collect the renormalization factors Z_f , Z_g , and Z_J to the orders of our interest. The results are given in Eq. (12).

5. The beta functions

After obtaining the renormalization factors, we can calculate the beta functions, defined in Eq. (13).

Taking the μ derivative of Eqs. (9,11) at the fixed bare couplings, we have

$$\begin{aligned} 0 &= \frac{\beta(g)}{g} + \beta(g) \frac{\partial \ln(Z_f^{-1} Z_g)}{\partial g} \\ &\quad + \beta(J) \frac{\partial \ln(Z_f^{-1} Z_g)}{\partial J} + \frac{\epsilon}{2}, \\ 0 &= \frac{\beta(J)}{J} + \beta(g) \frac{\partial \ln(Z_f^{-1} Z_J)}{\partial g} \\ &\quad + \beta(J) \frac{\partial \ln(Z_f^{-1} Z_J)}{\partial J} + \epsilon'. \end{aligned} \quad (\text{A27})$$

After some algebra, and setting $\epsilon' = 0$, we obtain the beta functions of g and J as in Eq. (14).

6. Renormalizability

We close this appendix by commenting on two issues regarding renormalizability. Our perturbative results for the self-energy and vertex corrections as well as the renormalization factors contain double poles such as $1/\epsilon^2$, $1/(\epsilon')^2$ and $1/\epsilon\epsilon'$. However, we find that terms of the form $\frac{1}{\epsilon} \ln \frac{\mu}{-\omega}$ all cancel out after we carry out expansions such as

$$\left(\frac{\mu}{-\omega} \right)^\epsilon = 1 + \epsilon \ln \frac{\mu}{-\omega} + O(\epsilon^2). \quad (\text{A28})$$

This is in accordance with the requirement of renormalizability [29,30,31,32]. In addition, we find that the double poles in the renormalization factors cancel out in the beta functions. This is also a consistency check for the minimal subtraction procedure, and is in accordance with the requirement that beta functions are analytic.

APPENDIX B: CALCULATIONS OF THE LOCAL SPIN SUSCEPTIBILITY

1. The local susceptibility to order ϵ^2

The local spin correlation function is defined in Eq. (17). We find it convenient to carry out our calculation in τ space. (The self-energy and vertex correction diagrams in Appendix A were calculated in ω space.) So we take the expressions for the boson and conduction electron propagators as

$$\begin{aligned} G_\phi^0(\tau) &= \tilde{K}_0^2 \left(\frac{\pi/\beta}{\sin \frac{\pi\tau}{\beta}} \right)^{2-\epsilon} \\ G_c^0(\tau) &= \tilde{N}_0 \left(\frac{\pi/\beta}{\sin \frac{\pi\tau}{\beta}} \right)^{1-\epsilon'} \end{aligned} \quad (\text{B1})$$

where \tilde{K}_0 and \tilde{N}_0 can be determined from K_0 and N_0 by

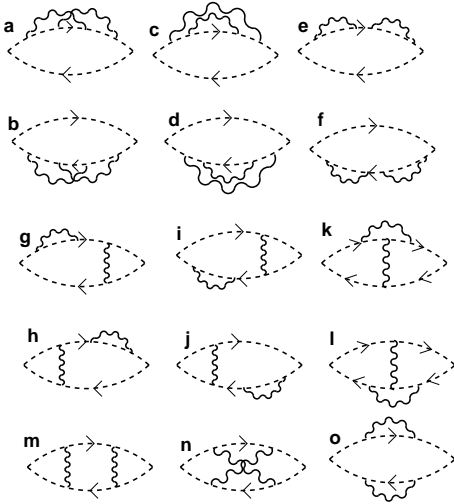


FIG. 12. Diagrams for the local spin susceptibility to the g^4 order.

$$\begin{aligned}\tilde{K}_0^2 &= K_0^2 \Gamma(2 - \epsilon) \\ \tilde{N}_0^2 &= N_0^2 \frac{\Gamma(2 - 2\epsilon')}{\Gamma^2(1 - \epsilon')},\end{aligned}\quad (\text{B2})$$

where $\Gamma(x)$ is the Euler Gamma function.

We now calculate the bare spin susceptibility in terms of the bare Hamiltonian (instead of the renormalized Hamiltonian used in Appendix A). The zero-th order result is given by

$$\begin{aligned}\tilde{\chi}_B^{(0)}(\tau) &= -\frac{1}{2} \int_0^\beta d\tau G_f^0(\tau) G_f^0(-\tau) = \frac{1}{2} e^{-\beta\lambda} \\ \chi_B^{(0)}(\tau) &= \lim_{\lambda \rightarrow \infty} \frac{1}{2} e^{\beta\lambda} \tilde{\chi}_B^{(0)}(\tau) = \frac{1}{4},\end{aligned}\quad (\text{B3})$$

where $G_f^0(\tau)$ is the bare f -electron propagator. This result is simply $S(S+1)/3$ with $S = 1/2$.

To the order g^2 and J^2 , the diagrams have been shown in Fig. 8 of Ref. [18]. Using the dimensional regularization adopted here, we find

$$\begin{aligned}\chi_B^{(2)}(\tau) &= -\frac{1}{4} \frac{2(K_0 g_B)^2 \tau^\epsilon}{\epsilon} \left(1 + C\epsilon + \frac{1}{2}(C^2 + \frac{\pi^2}{6})\epsilon^2 \right) \\ &\quad - \frac{1}{4} \frac{(N_0 J_B)^2 \tau^{2\epsilon'}}{2\epsilon'} (1 + \frac{\pi^2}{6}\epsilon'^2),\end{aligned}\quad (\text{B4})$$

where $C = 0.577215 \dots$ is the Euler Gamma constant.

To g^4 order, the diagrams are shown in Fig. 12. (Structurally, they are similar to those considered in ref. [24].) The result is

$$\begin{aligned}\chi_B^{(4)}(\tau) &= \frac{1}{4} \left(\frac{4}{\epsilon^2} + \frac{1+8C}{\epsilon} + 2 + 2C + 8C^2 + \frac{\pi^2}{3} \right) \times \\ &\quad \times (K_0 g_B)^4 \tau^{2\epsilon}\end{aligned}\quad (\text{B5})$$

We now express χ_B in terms of the renormalized coupling constants, g and J , by using Eqs. (9,11) in combination with our results for the renormalization factors,

Eq. (12). We can then determine the renormalization factor Z , defined in Eq. (18), by demanding that all the pole terms in χ_B are removed by those of Z^{-1} so that the renormalized susceptibility, $\chi = Z^{-1}\chi_B$, is regular. The result is

$$\begin{aligned}Z^{-1} &= 1 + \frac{2(K_0 g)^2}{\epsilon} + \frac{(N_0 J)^2}{2\epsilon'} \\ &\quad + \frac{4(K_0 g)^4}{\epsilon^2} - \frac{(K_0 g)^4}{\epsilon}.\end{aligned}\quad (\text{B6})$$

Eq. (19) then follows.

The renormalized susceptibility, $\chi = Z^{-1}\chi_B$, is equal to

$$\chi(\tau) = \chi^{(0)} [A(g, J, \epsilon) - B(g, J, \epsilon) \ln(\mu\tau) + O(\ln^2(\mu\tau))], \quad (\text{B7})$$

where, to the ϵ^2 order,

$$\begin{aligned}A(g, J, \epsilon) &= 1 - 2(K_0 g)^2(C + \frac{1}{2}(C^2 + \frac{\pi^2}{6})\epsilon) \\ &\quad + (K_0 g)^4(2 + 2C + 4C^2 - \frac{\pi^2}{3}) \\ B(g, J, \epsilon) &= 2(K_0 g)^2(1 + C\epsilon) + (N_0 J)^2 - 2(K_0 g)^4(1 + 4C).\end{aligned}\quad (\text{B8})$$

At the critical point (“C” in Fig. 1), the spin correlation has the form

$$\chi(\tau) = \chi^{(0)} A(g^*, J^*, \epsilon) \frac{1}{(\mu\tau)^\eta}, \quad (\text{B9})$$

where the exponent η can either be calculated from the renormalization factor Z in Eq. (21), or by $B(g^*, J^*, \epsilon)/A(g^*, J^*, \epsilon)$, which also yields $\eta = \epsilon$. The renormalized amplitude factor $A(g^*, J^*, \epsilon)$ is given by

$$A(g^*, J^*, \epsilon) = 1 - C\epsilon + \left(\frac{1}{2} + \frac{1}{4}C + \frac{1}{2}C^2 - \frac{1}{6}\pi^2 \right) \epsilon^2. \quad (\text{B10})$$

For large values of ϵ , it would require an appropriate resummation such as Padé approximation.

For the local moment fixed point, the local spin correlation has the form

$$\begin{aligned}\chi(\tau) &= \chi^{(0)} [A_L(\epsilon) - B_L(\epsilon) \ln(\mu\tau) + O(\ln^2(\mu\tau))], \\ A_L(\epsilon) &= A(g, J, \epsilon)|_{g=g_L^*, J=J_L^*=0} \\ &= 1 - C\epsilon + \left(\frac{1}{2} + \frac{1}{2}C^2 - \frac{1}{6}\pi^2 \right) \epsilon^2, \\ B_L(\epsilon) &= B(g, J, \epsilon)|_{g=g_L^*, J=J_L^*=0} \\ &= \epsilon - C\epsilon^2.\end{aligned}\quad (\text{B11})$$

Again, $B_L(\epsilon)/A_L(\epsilon) = \epsilon$ to the ϵ^2 order.

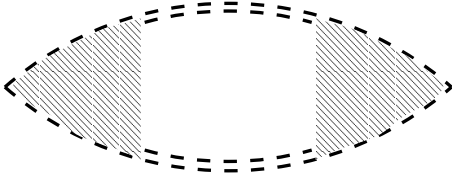


FIG. 13. The full local spin correlation function. Each double line represents the full f -electron propagator, while a shaded area denotes the full vertex.

2. The local susceptibility to all orders in ϵ

We now discuss the contributions to χ , to all orders of the perturbation theory. We can group all the diagrams in a manner illustrated in Fig. 13. Here, each double line represents the full f -electron propagator, while a shaded area denotes the full vertex. The renormalization factor for the spin, Z , is the product of the renormalization factors for the full f -electron propagators and those for the vertices. The renormalization factor for a full f -electron propagator is equal to Z_f . By inspecting the diagrams for each vertex, it is straightforward to see that to each order, this vertex is identical to that of the g -vertex. As a result, each vertex contributes a factor Z_g^{-1} . Taking these two things together, we end up with Eq. (24), to all orders. As discussed in Section IV B, Eq. (24) then leads to Eqs. (26,27).

APPENDIX C: RG ANALYSIS FOR ANISOTROPIC CASES

In the anisotropic case, the diagrams are topologically the same as those in the isotropic case, except for different coupling vertices. In the following, we will simply list the results.

The renormalization factors are given as

$$Z_f = 1 - \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^2}{2} + \frac{(K_0 g_z)^2}{4} \right) - \frac{1}{\epsilon^2} \left(\frac{(K_0 g_{\perp})^4}{8} + \frac{3(K_0 g_{\perp})^2(K_0 g_z)^2}{8} - \frac{(K_0 g_z)^4}{32} \right) + \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^4}{8} + \frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{4} \right) - \frac{1}{\epsilon'} \left(\frac{(N_0 J_{\perp})^2}{8} + \frac{(N_0 J_z)^2}{16} \right), \quad (C1)$$

$$Z_{g_{\perp}} = 1 + \frac{(K_0 g_z)^2}{4\epsilon} + \frac{1}{\epsilon^2} \left(\frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{4} + \frac{(K_0 g_z)^4}{32} \right) - \frac{(K_0 g_{\perp})^4}{8\epsilon} + \frac{(N_0 J_z)^2}{16\epsilon'}, \quad (C2)$$

$$Z_{g_z} = 1 + \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^2}{2} - \frac{(K_0 g_z)^2}{4} \right) + \frac{1}{\epsilon^2} \left(\frac{3(K_0 g_{\perp})^4}{8} - \frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{8} + \frac{(K_0 g_z)^4}{32} \right) + \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^4}{8} - \frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{4} \right) + \frac{1}{\epsilon'} \left(\frac{(N_0 J_{\perp})^2}{8} - \frac{(N_0 J_z)^2}{16} \right), \quad (C3)$$

$$Z_{J_{\perp}} = 1 - \frac{N_0 J_z}{\epsilon'} + \frac{(K_0 g_z)^2}{4\epsilon} + \frac{(N_0 J_z)^2}{16\epsilon'} + \frac{(N_0 J_{\perp})^2 + (N_0 J_z)^2}{2\epsilon'^2} - \frac{(K_0 g_{\perp})^2(N_0 J_z)}{\epsilon'(\epsilon + \epsilon')} - \frac{(K_0 g_z)^2(N_0 J_z)}{4\epsilon\epsilon'} + \frac{1}{\epsilon^2} \left(\frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{4} + \frac{(K_0 g_z)^4}{32} \right) - \frac{(K_0 g_{\perp})^4}{8\epsilon} \quad (C4)$$

$$Z_{J_z} = 1 - \frac{N_0(J_{\perp}^2/J_z)}{\epsilon'} + \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^2}{2} - \frac{(K_0 g_z)^2}{4} \right) + \frac{1}{\epsilon'} \left(\frac{(N_0 J_{\perp})^2}{8} - \frac{(N_0 J_z)^2}{16} \right) + \frac{(N_0 J_{\perp})^2}{\epsilon'^2} - \frac{(K_0 g_z)^2(N_0 J_z)}{\epsilon'(\epsilon + \epsilon')} - \frac{(2(K_0 g_{\perp})^2 - (K_0 g_z)^2)(N_0 J_{\perp}^2/J_z)}{4\epsilon\epsilon'} + \frac{1}{\epsilon^2} \left(\frac{3(K_0 g_{\perp})^4}{8} - \frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{8} + \frac{(K_0 g_z)^4}{32} \right) + \frac{1}{\epsilon} \left(\frac{(K_0 g_{\perp})^4}{8} - \frac{(K_0 g_{\perp})^2(K_0 g_z)^2}{4} \right) \quad (C5)$$

Eqs. (C1)- (C5) reduce to the isotropic results, Eq. (12), when we set $g_z = g_{\perp}$, $J_z = J_{\perp}$. Similar to the isotropic case, we obtain the beta functions for each coupling constant from these renormalization factors, which are given in Eqs. (32)- (35). We can then set $g_z = 0$ ($g_{\perp} = 0$) to discuss the xy (Ising) case.

We now turn to the calculation of the local spin susceptibility, focusing on the xy case only. Like in the isotropic case, we use the bare Hamiltonian. The contributions from the perturbative diagrams to the g^2 , J^2 , and g^4 orders are

$$\chi_B^{+- (2)}(\tau) = -\frac{1}{4} \frac{(K_0 g_{\perp}^B)^2 + (K_0 g_z^B)^2}{\epsilon} \tau^{\epsilon} \times \left(1 + C\epsilon + \frac{1}{2}(C^2 + \frac{\pi^2}{6})\epsilon^2 \right) - \frac{(N_0 J_{\perp}^B)^2 + (N_0 J_z^B)^2}{16\epsilon'} \tau^{2\epsilon'} (1 + \frac{\pi^2}{6}\epsilon'^2), \quad (C6)$$

$$\chi_B^{+- (4)}(\tau) = \frac{1}{4} \frac{(K_0 g_\perp^B)^4 + (K_0 g_\perp^B)^2 (K_0 g_z^B)^2}{2} \tau^{2\epsilon} \times \left(\frac{4}{\epsilon^2} + \frac{1+8C}{\epsilon} + 2 + 2C + 8C^2 + \frac{\pi^2}{3} \right).$$

(There are no contributions at the J or $g^2 J$ order.) Following the same RG procedure as in Appendix B, we can calculate the renormalization factor Z :

$$Z = 1 - \frac{1}{\epsilon} ((K_0 g_\perp)^2 + (K_0 g_z)^2) + \frac{1}{\epsilon} \left(\frac{(K_0 g_\perp)^4}{2} + \frac{(K_0 g_\perp)^2 (K_0 g_z)^2}{2} \right) - \frac{1}{\epsilon'} \left(\frac{(N_0 J_\perp)^2}{4} + \frac{(N_0 J_z)^2}{4} \right), \quad (C7)$$

and the anomalous dimension η at the critical point:

$$\eta = \frac{d \log Z}{d \log \mu} \Big|_{g_i=g_i^*, J_i=J_i^*} = (K_0 g_\perp^*)^2 + (K_0 g_z^*)^2 - (K_0 g_\perp^*)^4 - (K_0 g_\perp^*)^2 (K_0 g_z^*)^2 + \frac{(N_0 J_\perp^*)^2}{2} + \frac{(N_0 J_z^*)^2}{2} \quad (C8)$$

Setting $g_z = 0$ (for the xy case), the exponent becomes

$$\eta = (K_0 g_\perp^*)^2 - (K_0 g_\perp^*)^4 + \frac{(N_0 J_\perp^*)^2 + (N_0 J_z^*)^2}{2}. \quad (C9)$$

The renormalized amplitude $A(g, J, \epsilon)$ is given by

$$A(g, J, \epsilon) = 1 - (K_0 g_\perp)^2 (C + \frac{1}{2}(C^2 + \frac{\pi^2}{6})\epsilon) + (K_0 g_\perp)^4 (1 + C + 2C^2 - \frac{\pi^2}{6}) \quad (C10)$$

At the critical point, taking account of Eq. (37), we find

$$\eta = \epsilon, \quad A_C(\epsilon) = 1 - C\epsilon + \left(1 + \frac{3}{8}C + \frac{3}{2}C^2 - \frac{1}{4}\pi^2 \right) \epsilon^2. \quad (C11)$$

Near the bosonic stable fixed point where $J_\perp^* = J_z^* = 0$, which is given by Eq. (39), the results are

$$\eta_L = (K_0 g_\perp^*)^2 - (K_0 g_\perp^*)^4 = \epsilon, \quad A_L(\epsilon) = 1 - C\epsilon + \left(1 + \frac{3}{2}C^2 - \frac{1}{4}\pi^2 \right) \epsilon^2. \quad (C12)$$

APPENDIX D: KINK-GAS ANALYSIS OF THE ISING CASE

We now turn to the Ising case, by setting $g_\perp = 0$. The mapping to the kink-gas action is similar to that

given in ref. [21]. The resulting action, describing a one-dimensional long-ranged statistical-mechanical model, is similar to Eq. (7) of ref. [21] and, in our notation, takes the following form:

$$S(\tau_{2n}, \dots, \tau_1) = -2n \ln(y_j) + \sum_l (-1)^l h (\tau_{l+1} - \tau_l) / \tau_0 + \sum_{l < m} (-1)^{l+m} [2\kappa_j \ln(\tau_m - \tau_l) / \tau_0 + K(\tau_m - \tau_l)] \quad (D1)$$

where $[\tau_{2n}, \dots, \tau_1]$, for $n = 1, 2, \dots$, labels a sequence of spin flips (kinks) along the imaginary time axis, and y_j and κ_j are defined in Eq. (40). The last term, $K(\tau)$, originates from the g_z -coupling and takes the form:

$$K(\tau) = \kappa_g [(\tau/\tau_0)^\epsilon - 1] / \epsilon \quad (D2)$$

where κ_g is also specified by Eq. (40). The RG equations for this kink-gas problem has already been derived in ref. [21] and are given as follows:

$$\begin{aligned} \beta(y_j) &= -y_j(1 - \kappa_j - \kappa_g/2) \\ \beta(\kappa_j) &= 4\kappa_j y_j^2 \\ \beta(\kappa_g) &= -\kappa_g(\epsilon - 4y_j^2) \\ \beta(h) &= -h(1 - 2y_j^2) \end{aligned} \quad (D3)$$

This procedure is valid for arbitrary values of the stiffness constants κ_j and κ_g , provided the fugacity y_j is small.

An unstable fixed point still exists. To the linear order in ϵ , it is located at

$$\begin{aligned} (y_j^*)^2 &= \epsilon/4 \\ \kappa_g^* &= 2 \\ \kappa_j^* &= 0 \end{aligned} \quad (D4)$$

From Eq. (D3), we can easily see that κ_g is in fact irrelevant around this fixed point. We can then determine the critical exponents by staying within the $y_j - \kappa_g$ plane: the projection of the RG flow on the $y_j - \kappa_g$ plane is given in Fig. 4. Within the ϵ -expansion, then, the critical exponents of the unstable fixed point become identical to their counterparts for a similar fixed point of the classical ferromagnetic Ising chain with an long-range interaction that decays in distance in terms of a power-law exponent $2 - \epsilon$ [41]. The critical exponent η , for instance, can be straightforwardly calculated by combining Eq. (D3) and Eq. (D4). The result is $\eta = \epsilon$, the same value as it takes at the critical point of the corresponding long-ranged Ising chain [41].

[1] G. R. Stewart, Rev. Mod. Phys. **73**, 797 (2001).

- [2] H. v. Löhneysen *et al.*, Phys. Rev. Lett. **72**, 3262 (1994); H. v. Löhneysen, J. Magn. Magn. Mater. **200**, 532 (1999).
- [3] N. D. Mathur *et al.*, Nature **394**, 39 (1998); F. M. Grosche *et al.*, J. Phys.: Condens. Matter **13**, 2845 (2001).
- [4] O. Trovarelli *et al.*, Phys. Rev. Lett. **85**, 626 (2000).
- [5] K. Heuser *et al.*, Phys. Rev. B **57**, R4198 (1998).
- [6] S. Raymond, L. P. Regnault, J. Flouquet, A. Wildes, and P. Lejay, J. Phys.: Condens. Matter **13**, 8303 (2001).
- [7] V. A. Sidorov *et al.*, cond-mat/0202251.
- [8] P. Estrela, A. de Visser, T. Naka, F. R. de Boer, and L. C. J. Pereira, cond-mat/0009324.
- [9] C. L. Seaman *et al.*, Phys. Rev. Lett. **67**, 2883 (1991); M. B. Maple *et al.*, J. Low Temp. Phys. **95**, 225 (1994).
- [10] B. Andraka and A. M. Tsvelik, Phys. Rev. Lett. **67**, 2886 (1991).
- [11] M. C. Aronson *et al.*, Phys. Rev. Lett. **75**, 725 (1995).
- [12] O. O. Bernal, D. E. MacLaughlin, H. G. Lukefahr, and B. Andraka, Phys. Rev. Lett. **75**, 2023 (1995).
- [13] A. Schröder *et al.*, Nature **407**, 351 (2000).
- [14] A. Schröder, G. Aeppli, E. Bucher, R. Ramazashvili, and P. Coleman, Phys. Rev. Lett. **80**, 5623 (1998).
- [15] O. Stockert, H. v. Löhneysen, A. Rosch, N. Pyka, and M. Loewenhaupt, Phys. Rev. Lett. **80**, 5627 (1998).
- [16] A. Rosch, A. Schröder, O. Stockert, and H. v. Löhneysen, Phys. Rev. Lett. **79**, 159 (1997).
- [17] Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, Nature **413**, 804 (2001).
- [18] Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, cond-mat/0202414.
- [19] Q. Si, J. L. Smith, and K. Ingersent, Int. J. Mod. Phys. B **13**, 2331 (1999).
- [20] For a recent review on related issues see P. Coleman, C. Pépin, Q. Si, and R. Ramazashvili, J. Phys.: Condens. Matter **13**, R723 (2001).
- [21] Q. Si and J. L. Smith, Phys. Rev. Lett. **77**, 3391 (1996).
- [22] J. L. Smith and Q. Si, cond-mat/9705140; Europhys. Lett. **45**, 228 (1999).
- [23] A. M. Sengupta, cond-mat/9707316; Phys. Rev. B **61**, 4041 (2000).
- [24] M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000).
- [25] S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
- [26] A. Georges, O. Parcollet, S. Sachdev, Physical Review B **63**, 134406 (2001).
- [27] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge Univ. Press, Cambridge, 1993).
- [28] A. A. Abrikosov, Physics **2**, 5 (1965).
- [29] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, (Oxford University Press, London, 1996), Chap. 11.
- [30] E. Brézin, J. C. Le Guillou, and J. Zinn-Justin, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, London, 1976), Vol. 6.
- [31] D. J. Gross, in *Methods in Field Theory*, edited by R. Balian and J. Zinn-Justin, (North-Holland Publishing, Amsterdam, 1975), p. 141.
- [32] G. 't Hooft, Nucl. Phys. B **61**, 455 (1973); G. 't Hooft and M. Veltman, *ibid.* **44**, 189 (1972).
- [33] We adopt the sign convention of the field-theory approach.
- [34] Here we speak of phases having in mind the one-dimensional long-ranged statistical-mechanical model associated with the Bose-Fermi Kondo model.
- [35] M. E. Fisher, S.-K. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972).
- [36] M. Suzuki, Prog. Theor. Phys. **49**, 424, 1106, 1440 (1973).
- [37] H. Spohn and R. Dumche, J. Stat. Phys. **41**, 389 (1985) and references therein.
- [38] J. L. Smith and Q. Si, Phys. Rev. B **61**, 5184 (2000).
- [39] R. Chitra and G. Kotliar, Phys. Rev. Lett. **84**, 3678 (2000).
- [40] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Rev. Mod. Phys. **68**, 13 (1996).
- [41] J. M. Kosterlitz, Phys. Rev. Lett. **76**, 1577 (1976).
- [42] L. Zhu and Q. Si, Bull. Am. Phys. Soc. **47**(1), 89 (2002)[March].
- [43] G. Zarand, private communication.